Quantum statistics

So far we had tried to describe the statistical dynamics of a large number of particles, each of which obey laws of classical (Newtonian) mechanics. However it is known that quantum mechanics is the law of the nature. They are expected to obey uncertainty principle. Thus the factor h^3 which we assumed to be the smallest volume in the phase space is in fact truly the smallest volume in phase space. Its value is decided by the uncertainty relation

$$\Delta x \cdot \Delta p \geq \hbar \tag{1}$$

Here $\hbar = \frac{h}{2\pi}$, with $h = 6.63 \times 10^{-34}$Joules sec, being the Planck's constant. Hence we should have taaken the smallest phase space volume to be really $\hbar^3$.

Rules of quantum mechanics has more consequences than just the uncertainty. According to quantum mechanics, every particle will be in a state described by a wavefunction. This wavefunction will have a finite spread in the space given by the de Broglie wavelength. Two particles will interact if their wavefunctions overlap with each other. The de Broglie wavelength of an ideal gas molecule can be easily estimated to be

$$\lambda = \frac{h}{\sqrt{mk_B T}} \tag{2}$$

The above wavelength is often called the *thermal wavelength*. If any particle comes closer than the above distance to a particle of the above de Broglie wavelength, they would start interacting, as per quantum mechanics. Clearly, we can define a cubic cell of volume

$$\lambda^3 = \left(\frac{h^2}{mk_B T}\right)^{\frac{3}{2}} \tag{3}$$

inside which the interaction between two particles will be significant according to quantum mechanics.